

Who Benefits from Platform Exclusivity?

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Abstract

A vertically integrated platform plays a dual role. It runs a marketplace for third-party products and sells its own product on that marketplace. This paper asks whether such a platform should keep its product exclusive or also sell it on a rival platform, and how the choice affects consumers. I build a duopoly model in which each platform has a differentiated first-party product. A platform can allow its rival to sell its first-party product at a commission rate determined by bargaining. Both platforms give up exclusivity in equilibrium to increase profit, whether consumers are singlehoming or multihoming. Intuitively, the commission softens competition between the platforms because each platform benefits from the sales of both products, which gives both an incentive to raise prices. Consumer surplus is lower without exclusivity when consumers can switch between platforms.

1 Introduction

Two-sided platforms bring together two groups of users, such as buyers and sellers of some underlying product. For example, video game consoles such as the Sony PlayStation and the Microsoft Xbox provide the platform on which consumers play video games provided by content developers. In such markets, products may or may not be exclusive. For example, from its release in 2018 until 2022, the popular game “God of War” (developed by Santa Monica Studio) was available only on the PlayStation. In contrast, the very popular Fortnite game (developed by Epic Games) has long been available on the PlayStation, the Xbox and other platforms.

In recent years, platform providers in a variety of markets have opted for a vertical integration strategy in which they produce content for sale on the platform. For example, video platforms such as Amazon Prime Video, Apple TV, and Netflix all produce much of the video content they show. In the video game market, both Microsoft and Sony have acquired multiple previously independent game developers. One possible explanation for such vertical integration is that exclusivity might be easier to achieve and maintain. However, integrated platforms do not always opt for exclusivity and sometimes make their products available on other platforms, as in the video game market.

The primary research question of this paper is when integrated platforms choose to sell their products on an exclusive basis and when they choose to “cross-sell,” that is, to sell on the rival platform as well. In addition, I examine the comparative welfare properties (as measured by consumer and producer surplus) of exclusivity and cross-selling.

To answer this question, I build a tractable duopoly model with two differentiated products and two platforms, each integrated with a content provider, which can adopt a cross-selling strategy by giving up exclusivity and selling on the rival’s platform. The platforms play a three-stage game. In the first stage, each firm decides whether to cross-sell. The second stage is a Nash bargaining stage in which, if cross-selling occurs, the firms bargain over the commission. The third stage is a Bertrand pricing stage. While the model is sufficiently general to apply to many platform markets, it is motivated by the video game market.

I study two cases. First, when consumers cannot move across platforms (singlehoming), if one platform, say platform 1, sells exclusively, consumers on the other platform do not have access to platform 1’s content. Second, when consumers can move across platforms (multihoming), they can access exclusive content on the other platform by paying an additional constant console fee. In each case there are three potential equilibriums: (i) an exclusive equilibrium, in which both firms sell exclusively, (ii) a full-cross-selling equilibrium, in which both firms cross-sell, and (iii) a semi-cross-selling equilibrium, in which one firm cross-sells and the other

sells exclusively.

In the traditional antitrust view, one might think that vertical integration brings input foreclosure and thus makes exclusivity more likely. However, the results show that in both cases the optimal first-stage strategy for both firms is to give up exclusivity and supply their rival, so the three-stage game ends in a full-cross-selling equilibrium. The main driver of this strategy is not the extra revenue from selling on the other platform, since in this equilibrium both firms agree on a 100% commission, which leaves no revenue to the product owner. I explain this result from the perspective of collusion. After vertical integration, the trade-off of product exclusivity changes dramatically for both the platforms and the content providers. A bargain between a platform and a content provider is transformed into a profit split between platforms, which gives the platforms an incentive to push up the market price together, using the commission as a tool.

This non-exclusivity can have an anti-competitive effect. When the console fee is below its upper limit, the full-cross-selling equilibrium generates a consumer surplus higher than that of the singlehoming exclusive equilibrium but lower than that of the multihoming exclusive equilibrium. One potential explanation is that the exclusive equilibrium under singlehoming works as a monopoly market, while under multihoming it works as a competitive market. The higher price generated by the 100% commission rate harms consumer welfare.

As for other equilibrium characteristics, under singlehoming, firms' equilibrium behavior depends only on the degree of differentiation between the two products. As substitutability increases, the negotiated commission increases, and it approaches 1 as the products become perfect substitutes. In the full-cross-selling and semi-cross-selling equilibriums, higher substitutability reduces profits. Under multihoming, firms' behavior depends not only on the degree of content differentiation but also on the console fee. The negotiated commission is larger when substitutability and the console fee are larger. The equilibrium profits in all three cases decrease with substitutability.

I further extend the analysis by replacing the quadratic demand system with general demand functions that satisfy several essential assumptions, and I prove that most of the findings still hold under these alternative demands.

This paper is organized as follows. Section 2 reviews the related literature and the industry that motivates the model. Section 3 sets out the baseline model, explaining how the game is played, how consumers react to the market, and how the firms interact with each other under different strategies. Sections 4 to 6 solve the three stages of the game by backward induction. Section 7 examines the competitive impact under a general demand system, and Section 8

concludes.

2 Background

2.1 Related Literature

This paper studies how platforms' profits and consumer welfare are affected by their exclusivity strategies when they are vertically integrated with content providers. It is related to several streams of the industrial organization literature, but above all it is motivated by the seminal papers on two-sided markets (Rochet and Tirole, 2006; Caillaud and Jullien, 2003).

This paper is related to the large theoretical literature on exclusivity. Though modeled in various ways, exclusive contracts have proved to be a powerful tool for platforms to manage homing decisions on the two sides of the market. How the platforms and content providers divide the overall profit plays a crucial role in determining the equilibrium outcome. Armstrong (1999) provides one of the first studies of exclusive contracts in media platforms. With independent content providers and platforms financed only through subscription fees, he finds that a lump-sum payment for content favors exclusive contracts more than a per-subscriber fee does. Similarly, Harbord and Ottaviani (2001) show that content providers find it most profitable to sell exclusively for a lump-sum payment, while it is optimal for the platform to use a per-subscriber fee. Hagiu and Lee (2011) build a model with two platforms and a continuum of content providers, in which platforms can offer content providers transfers contingent on whether they join exclusively. They specify two ways in which content providers and platforms set prices and divide revenue, depending on who controls pricing. Their equilibrium result shows that content providers' exclusivity decisions ultimately depend on who controls the retail price. If content providers sell their content outright and give up control, they tend to sell exclusively.

Beyond the exclusivity research, another stream of literature studies how vertical integration changes the exclusivity decisions of platforms and content providers. Weeds (2016) shows that vertically integrated platforms have an incentive to withhold premium content from their rivals as long as there is some form of scale effect such that the asymmetry induced by content exclusivity raises industry profits. By contrast, D'Annunzio (2017) finds that premium content is always provided exclusively, with or without vertical integration, yet vertically integrated platforms have stronger incentives to invest in the quality of their exclusive content.

Carroni et al. (2018) focus on superstar products and find that exclusivity is less likely to occur under vertical integration than under vertical separation. Their explanation lies in the trade-off created by vertical integration. After acquiring the product, the platform's loss from

giving up exclusivity is small compared with the gain from the larger demand it faces when selling the product on its rival's platform.

While the rich theoretical literature studies how firms act in equilibrium, empirical papers focus more on the competitive impact of vertical integration. Foreclosure is one major reason why vertical integration is thought to be anti-competitive, but the empirical evidence is still far from definitive. Hortagsu and Syverson (2007) estimate its impact using a rich data set of cement and ready-mixed concrete plants. Their results show that a greater presence of vertically integrated firms in a market comes with lower prices and higher quantities, suggesting no anti-competitive foreclosure effect. Lee (2013) studies exclusivity in the video game industry and concludes that exclusivity can increase sales and consumer welfare by favoring entrant platforms.

Some researchers believe that vertical integration can also have pro-competitive effects. As summarized by Slade (2022), the dominant reason is the elimination of successive monopoly markups, or double marginalization (EDM). Crawford et al. (2018) estimate a structural model of the regional sports network market. They find that vertical integration has a significant efficiency-enhancing effect through reduced double marginalization and increased carriage of channels, alongside potential welfare harm from foreclosure. However, as Salinger (1991) shows theoretically, EDM may not be efficient when the integrated firms produce multiple products. Luco and Marshall (2020) show evidence from the carbonated-beverage industry that when the Coca-Cola Company and PepsiCo acquired some of their bottlers, the prices of their own products decreased. However, the prices of the other products sold by the integrated bottlers increased, making the overall competitive effect ambiguous.

This paper is also related to a newer literature that studies platforms' preferences over their roles under different conditions and the impact of those roles on platform competition. Hagiu and Wright (2015) recognize that platforms can choose between functioning as a marketplace and as a reseller. The fundamental distinction lies in the allocation of control rights over non-contractible decisions such as prices and advertising. The optimal mode in their model is driven by market characteristics such as the relative importance of suppliers' and the platform's private information, spillover effects of marketing activities, and product features. Hagiu et al. (2022) further show that the dual mode (acting as both marketplace and seller) is always better for the platform than either single role. Moreover, a ban on the dual mode can harm consumers and decrease overall welfare.

2.2 The Video Game Industry

With revolutionary developments in graphics programming, video games have evolved from mechanical arcade cabinets to modern home consoles, from pixelated figures to cinematic scenery, and from a coin-operated pastime of children and hobbyists to a \$269 billion worldwide industry spanning software, hardware, social media, and e-sports. Today, 67% of American adults identify as gamers, and 75% of Americans have at least one video game console in their household (Tripp et al., 2020).

The specific type of video game analyzed in this paper is the console game. A typical console game system comprises a hardware platform (the console) and software (the games). Each platform works as a two-sided market with game developers on one side and gamers on the other. Consumers pay for the console and for each game they want to play.

Today, Nintendo, Sony, and Microsoft make up the whole console market. They are allowed to, and enthusiastic about, running their own game studios and acquiring game developers. Any title produced or owned by the platform is called a first-party title, while the others are called third-party titles, whose developers pay a royalty for every copy sold. Since consoles have little, if any, stand-alone utility for consumers, the platforms invest heavily in their first-party titles and make them exclusive to attract gamers. At the same time, independent game developers normally do not have a size comparable to that of the first-party developers, so most top-selling games, the superstar games, are first-party exclusives. On Sony’s PlayStation 4, one of the three giant console platforms, 8 of the 10 top-selling games are first-party exclusives, while only 1 of the 20 top-selling games on the Nintendo Switch is a third-party title.

Recently, there has been a surge of acquisitions in the console game industry. All of the 10 largest mergers and acquisitions in this industry took place in the last six years, and 8 of them in the last three years. In September 2020, Microsoft acquired the video game studio ZeniMax for \$8.1 billion, an all-time record for a video game studio acquisition until 16 months later, when it spent another \$68.7 billion to acquire Activision Blizzard. Over the same period, Sony acquired 10 independent game studios in three years, the largest deal being worth \$3.7 billion. However, while most analysts believe that the two giants are allocating resources for an exclusivity war, both have stated that all games from the newly acquired studios will remain available on the other platforms.

3 Model Preliminaries

In this section, I consider a simplified model of the video game industry. Specifically, I consider a market with only two platforms, $i = 1, 2$, in which consumers can buy only the content they provide.

Consumers. On each platform there is a group of existing consumers with identical preferences. In most previous studies of two-sided markets, consumers enjoy the utility of the platform itself and of all the content provided as soon as they join the platform. This is not ideal for many of today's content platforms, for two main reasons. First, a hardware platform can hardly generate standalone utility for consumers. Imagine a Kindle without an e-book, an iPad without software, or a PlayStation 5 without a game. Without content they are simply useless to consumers. Second, consumers cannot enjoy most of the attractive content before they purchase it. Even for some free software and games, unlocking essential functions requires payment.

Therefore, I modify how consumers derive utility. They receive no standalone utility from joining the platform or from any fringe game, but only from purchasing the two differentiated superstar games on the market, game x and game y. This captures the fact that in practice, when consumers decide which platform to join, they focus primarily on the superstar products, and the fringe products are an extra bonus. Each consumer can purchase any game available on their platform, but can buy at most one unit of each game.

If both games are available on platform i , the demand for each product on each platform can be derived from the following symmetric quadratic utility function:

$$U_i = a(x_i + y_i) - \frac{1}{2}(x_i^2 + y_i^2) - sx_iy_i \quad (1)$$

where a is an exogenous parameter large enough to ensure positive demand, x_i and y_i are the demands for game x and game y on platform i , and s is the degree of substitutability between the two games. Here I assume the games are substitutes, so s is positive and cannot exceed 1, i.e., $0 < s < 1$. Note that this utility function implicitly represents the preferences of many consumers, since presumably each consumer buys at most one unit of each game. Let P^X and P^Y denote the prices of the two games. The inverse demand functions are:

$$P^X = a - x - sy \quad (2)$$

$$P^Y = a - y - sx \quad (3)$$

Platforms. Each of the two platforms is vertically integrated with one of the superstar games. Specifically, platform 1 owns game x, and platform 2 owns game y. Each platform sets the price only for its own game, which is sold at that price on any platform.

Because of the dual role of a vertically integrated platform, as seller and as marketplace, its revenue consists of two parts: the revenue from selling its own product and the revenue from selling the other firm’s product. In most real-world platform businesses, the platform earns a fixed amount or a fixed proportion from each transaction that takes place on it. In this model, since the focus is on “superstar” products that can attract subscriptions, the product owners are allowed to have bargaining power.

To be more specific, if one game is sold on the rival’s platform, the two platforms negotiate a commission rate $(1 - \sigma)$ owed to the selling platform, and the owner of the product earns the rest. For instance, if platform 1 adopts the cross-selling strategy while platform 2 does not, the revenues of the two platforms are:

$$\Pi_1 = P^X x_1 + \sigma P^X x_2 \quad (4)$$

$$\Pi_2 = (1 - \sigma) P^X x_2 + P^Y y_2 \quad (5)$$

The choice of the commission reflects the platforms’ preference between the two roles. If they end up selecting a small commission (a large σ), then, relatively, they earn more by selling their own products than by selling the other firm’s products, which suggests that the platforms value the seller role more.

For most digital content, such as games, movies, and series, the majority of the cost is incurred during production and development, and the cost of distributing one additional copy is negligible. Accordingly, the cost of the content is largely sunk, since the firm cannot undo it over the period of the game analyzed in this paper. Therefore, I treat the cost as irrelevant to the platform’s strategy and normalize it to zero. Since the cost is zero, revenue equals profit.

The equilibrium concept is a sequentially rational equilibrium of the following three-stage game:

1. Both platforms decide whether to adopt cross-selling.
2. If at least one platform cross-sells, the platforms engage in Nash bargaining over the commission. If they settle, they play on with the negotiated commission. If they fail to settle, no cross-selling takes place in the following stage.
3. The platforms play a Bertrand price game.

4 Stage 3: The Bertrand Price Game

In this section, I solve the third stage of the overall game, the Bertrand price game. I first analyze the case in which consumers cannot move across platforms, the singlehoming case. This case captures the real-world scenario in which few consumers change their hardware platform because of high console prices or brand loyalty. For example, most Windows PC users need to seriously consider whether switching to macOS is worth the high price of a MacBook, and some MacBook users will not switch to Windows because of their loyalty to Apple and macOS.

I then allow consumers to move across platforms (the multihoming case). The crucial change is that consumers can now purchase an exclusive game on the other platform by first purchasing that hardware platform. I discuss this case in detail below.

In both cases, there are three possible equilibriums:

- (i) Exclusive equilibrium: both firms sell their games exclusively.
- (ii) Full-cross-selling equilibrium: both firms sell their games on both platforms.
- (iii) Semi-cross-selling equilibrium: one platform gives up its exclusivity while the other keeps it. Without loss of generality, I call the firm that cross-sells firm 1 and the other firm 2.

4.1 The Singlehoming Case

Exclusive Equilibrium

If neither platform adopts the cross-selling strategy, each game is sold only on its own platform. Thus consumers on platform 1 do not have access to game y, and vice versa. Consumers' utilities simplify to:

$$\begin{aligned} U_1 &= ax_1 - \frac{1}{2}x_1^2 \\ U_2 &= ay_2 - \frac{1}{2}y_2^2 \end{aligned}$$

Using the budget constraints, we can derive the demands as follows:

$$\begin{aligned} x_1(P^X) &= a - P^X \\ y_2(P^Y) &= a - P^Y \end{aligned}$$

When both platforms sell exclusively, each makes profit only by selling on its own platform. Therefore, each platform is a monopoly. Using the demands, the platforms' profits are:

$$\Pi_1 = P^X (a - P^X)$$

$$\Pi_2 = P^Y (a - P^Y)$$

Each platform sets its price to maximize its profit. Solving the maximization problem gives the equilibrium prices of games x and y. Since both the utility functions and the profit functions are symmetric, we obtain one price for both games:

$$P_E^X = P_E^Y = \frac{a}{2} \quad (6)$$

The equilibrium profits are then:

$$\Pi_1 = \Pi_2 = \Pi_{NC} = \frac{a^2}{4} \quad (7)$$

Full-Cross-Selling Equilibrium

If both platforms adopt the cross-selling strategy, all consumers have access to both games, and their utilities are:

$$U_i = a(x_i + y_i) - \frac{1}{2}(x_i^2 + y_i^2) - sx_i y_i$$

Note that this case is a standard Bertrand model with quadratic utility. The demands are:

$$\begin{aligned} x_1 = x_2 &= \frac{1}{1-s^2} [a(1-s) - P^X + sP^Y] \\ y_1 = y_2 &= \frac{1}{1-s^2} [a(1-s) - P^Y + sP^X] \end{aligned}$$

When both firms cross-sell, each earns profit from three sources: (i) selling its own game on its own platform, (ii) selling its own game on the rival's platform, and (iii) selling the rival's game on its own platform. Formally,

$$\begin{aligned} \Pi_1 &= P^X x_1 + \sigma P^X x_2 + (1-\sigma) P^Y y_1 \\ \Pi_2 &= P^Y y_2 + \sigma P^Y y_1 + (1-\sigma) P^X x_2 \end{aligned}$$

Platforms 1 and 2 maximize their profits by setting the prices of their games. As in the exclusive equilibrium, the profit functions are symmetric, so in equilibrium there is one price for both games:

$$P_C^X = P_C^Y = \frac{(1-s)(1+\sigma)a}{1+\sigma-s} \frac{a}{2} \quad (8)$$

Since both s and σ lie between 0 and 1,

$$(1-s)(1+\sigma) = 1 + \sigma - s - s\sigma < 1 + \sigma - s \quad (9)$$

so the full-cross-selling equilibrium generates a price that is always lower than the exclusive equilibrium price.

Semi-Cross-Selling Equilibrium

Without loss of generality, I call the firm that cross-sells firm 1 and the other firm 2. Accordingly, both games are available on platform 2, but only game x on platform 1. The utilities of their consumers are:

$$U_1 = ax_1 - \frac{1}{2}x_1^2$$

$$U_2 = a(x_2 + y_2) - \frac{1}{2}(x_2^2 + y_2^2) - sx_2y_2$$

The demands are:

$$x_1 = a - P^X$$

$$x_2 = \frac{1}{1-s^2} [a(1-s) - P^X + sP^Y]$$

$$y_2 = \frac{1}{1-s^2} [a(1-s) - P^Y + sP^X]$$

In this case, the two firms earn profits from different sources. Firm 1 makes profit by selling its own game on both platforms, while firm 2 makes profit by selling its own game and firm 1's game on its own platform. The profits of the two platforms are:

$$\Pi_1 = P^X x_1 + \sigma P^X x_2$$

$$\Pi_2 = P^Y y_2 + (1-\sigma) P^X x_2$$

Since the demand for each game is decreasing in its own price, each firm's profit is single-peaked in its own price. The optimization problem is solved by the following first-order conditions:

$$\partial \Pi_1 / \partial P^X = 0 \quad ; \quad \partial \Pi_2 / \partial P^Y = 0$$

Equilibrium Comparison

To illustrate the results, I compute a numerical example with $a = 40$ and $s = 0.6$.

Among the three cases, an asymmetric equilibrium occurs only when one platform cross-sells and the other does not. Compared with the exclusive equilibrium, all other equilibriums generate lower prices and accordingly higher consumer surplus. As σ grows, the platforms value

the seller role more, which gives them a stronger motive to compete with lower prices. This result shows that the market is more competitive under cross-selling, in line with intuition, since consumers have more choices on both platforms.

The more novel result is how the platforms' profits change with the strategies and the commission level. If adopting the cross-selling strategy led to a more competitive market with lower profit, the platforms would have no motive to adopt it. However, the results show that this is not always the case.

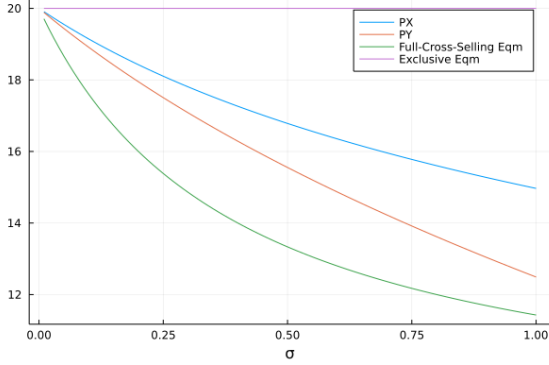


Figure 1: Price change

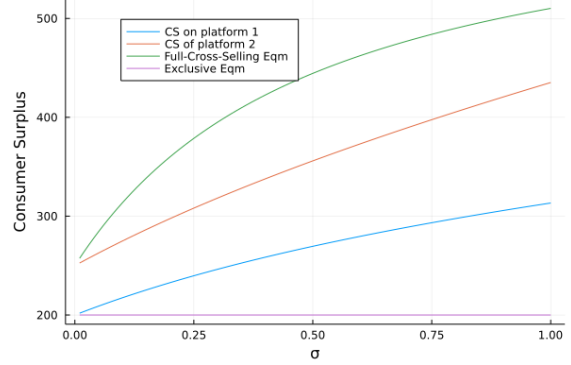


Figure 2: Welfare change

In Figure 3, the green line represents the profit in the full-cross-selling equilibrium, while the purple line is the profit in the exclusive equilibrium. The other two lines represent each platform's profit when firm 1 cross-sells and firm 2 does not. Intuitively, firm 1 benefits from a larger σ , since it plays the seller role on both platforms. By contrast, firm 2 prefers a small σ , so that it earns more from the commission.

It is also interesting that the full-cross-selling equilibrium profit decreases with σ . When σ is large and both firms cross-sell, the market is more competitive, with low prices and low profits. When σ goes to zero, however, each platform makes more profit from selling the other firm's product on its own platform than from selling its own product on the other platform, suggesting that the marketplace role is valued more.

I also compute a numerical example with $a = 40$ and $\sigma = 0.6$ to observe how the outcomes change with the degree of product differentiation. The trends for prices and consumer surplus are much the same as when σ changes. A larger s leads to lower prices and higher consumer surplus. The intuition is that when the products are more alike, the market becomes more competitive, which leads to low prices and high consumer surplus. What differs is how the equilibrium profits change with s . As shown in the figure, all equilibrium profits except the exclusive equilibrium profit decrease with s . The full-cross-selling equilibrium profit decreases faster than the semi-cross-selling equilibrium profit.

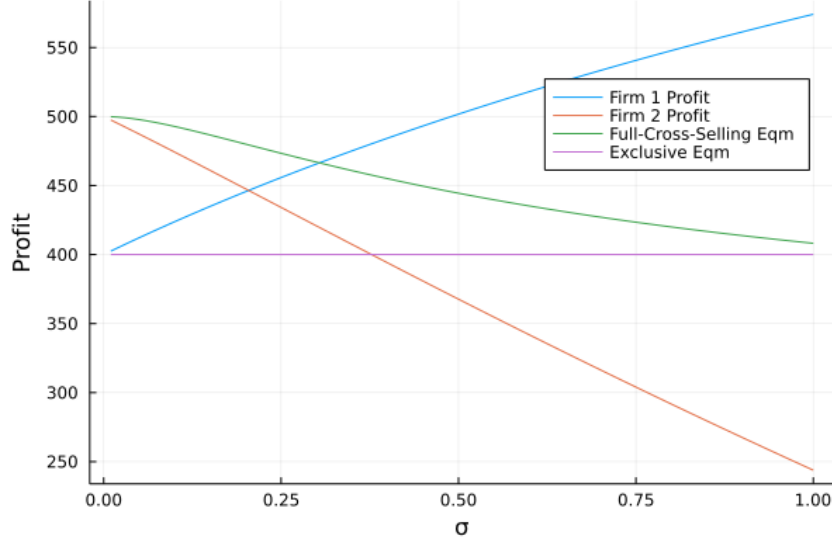


Figure 3: Profits change with commission

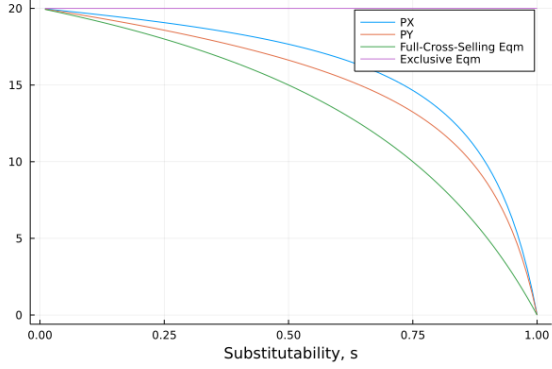


Figure 4: Price change

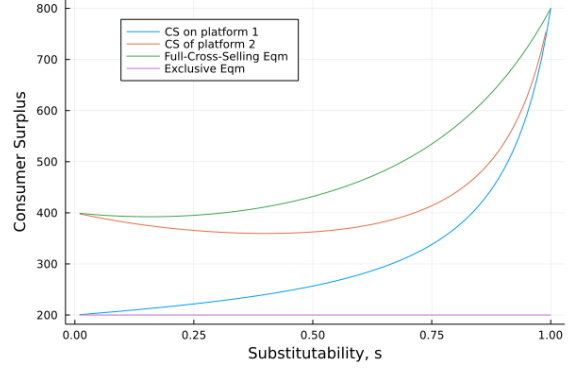


Figure 5: Welfare change

4.2 The Multihoming Case

In the multihoming case, I allow consumers to move across platforms by paying a console fee c , which is earned by the platform they move to. In this case, even if one platform does not cross-sell, consumers on the other platform still have access to its game. In practice, the subscription fee or console fee of a platform is usually stable over a long period and does not differ dramatically across platforms. Therefore, I assume that the console fee is a constant.

Since consumers can move across platforms, they have access to both games. In all cases they therefore have the same utility functions:

$$U_1 = a(x_1 + y_1) - \frac{1}{2}(x_1^2 + y_1^2) - sx_1y_1$$

$$U_2 = a(x_2 + y_2) - \frac{1}{2}(x_2^2 + y_2^2) - sx_2y_2$$

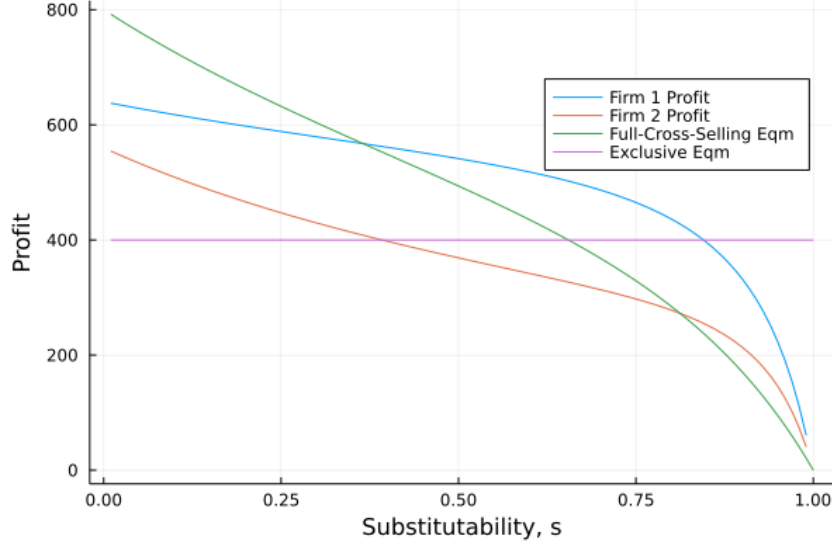


Figure 6: Profits change with substitutability

Exclusive Equilibrium

The difference among the cases lies in consumers' budget constraints and platforms' profits. When neither platform cross-sells, all consumers need to pay the console fee to access the game on the other platform. Let M_1 and M_2 be the numeraire consumption of the two groups of consumers. The budget constraints are:

$$M_1 \leq P^X x_1 + (P^Y + c) y_1$$

$$M_2 \leq (P^X + c) x_2 + P^Y y_2$$

Accordingly we have the demands as:

$$x_1 = \frac{1}{1 - s^2} [a(1 - s) - P^X + sP^Y + sc]$$

$$y_1 = \frac{1}{1 - s^2} [a(1 - s) - P^Y + sP^X - c]$$

$$x_2 = \frac{1}{1 - s^2} [a(1 - s) - P^X + sP^Y - c]$$

$$y_2 = \frac{1}{1 - s^2} [a(1 - s) - P^Y + sP^X + sc]$$

When there is no cross-selling, the platforms do not need to determine the commission $(1 - \sigma)$. They earn revenue from selling on both platforms plus the console fee. Their profits

are:

$$\Pi_1 = P^X (x_1 + x_2) + cx_2$$

$$\Pi_2 = P^Y (y_1 + y_2) + cy_1$$

In equilibrium, both platforms maximize their profits by choosing prices. Solving the first-order conditions, we have:

$$P_E^X = P_E^Y = \frac{1-s}{2-s}a - \frac{c}{2} \quad (10)$$

Compared with the singlehoming exclusive equilibrium price $P_E^X = P_E^Y = \frac{a}{2}$, with positive s and c the multihoming price is always lower.

Note that with the console fee, demands can be negative even at non-negative prices. Specifically, for the equilibrium demands y_1 and x_2 to be non-negative, we need:

$$c \leq \frac{2}{1+s} \frac{1-s}{2-s} a \quad (11)$$

To support a non-negative price, we need:

$$c \leq \frac{2(1-s)}{2-s} a \quad (12)$$

Since $\frac{2}{1+s} < 2$ for all $s \in (0, 1)$, not every non-negative price supports non-negative demands. Negative demands or prices are not feasible, so in the following analysis I consider only the combinations of s and c such that all demands and prices are positive.

Full-Cross-Selling Equilibrium

When both firms cross-sell, consumers have no need to move across platforms and pay the console fee. Therefore, this case is identical to the full-cross-selling equilibrium in the singlehoming case.

Semi-Cross-Selling Equilibrium

Without loss of generality, assume that platform 1 cross-sells. Then consumers on platform 1 need to pay the console fee to access game y , while consumers on platform 2 do not pay any console fee. Consumers' budget constraints are:

$$M_1 \leq P^X x_1 + (P^Y + c) y_1$$

$$M_2 \leq P^X x_2 + P^Y y_2$$

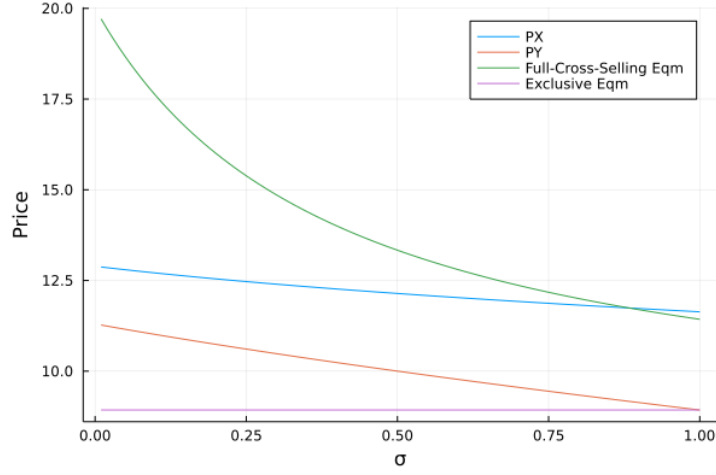


Figure 7: Price change with commission

The demand system is:

$$\begin{aligned}
 x_1 &= a - sy_1 - P^X \\
 y_1 &= a - sx_1 - P^Y - c \\
 x_2 &= a - sy_2 - P^X \\
 y_2 &= a - sx_2 - P^Y
 \end{aligned}$$

Platform 1 earns from selling game x on both platforms, while platform 2 earns from selling game y on both platforms, from selling game x on platform 2, and from the console fee. Their profits are:

$$\begin{aligned}
 \Pi_1 &= P^X (x_1 + \sigma x_2) \\
 \Pi_2 &= P^Y (y_1 + y_2) + (1 - \sigma) P^X x_2 + cy_1
 \end{aligned}$$

Solving the first-order conditions gives analytical solutions for P^X and P^Y . Since they are notationally complicated, I leave the detailed derivation and the solutions to the appendix.

Equilibrium Comparison

The multihoming case shares many trends with the singlehoming case. The equilibrium prices in the semi-cross-selling and full-cross-selling equilibriums decrease with σ while consumer surplus increases. One noticeable difference is that the exclusive equilibrium now has the lowest price and the highest consumer surplus, while the full-cross-selling equilibrium has the highest price and the lowest consumer surplus.

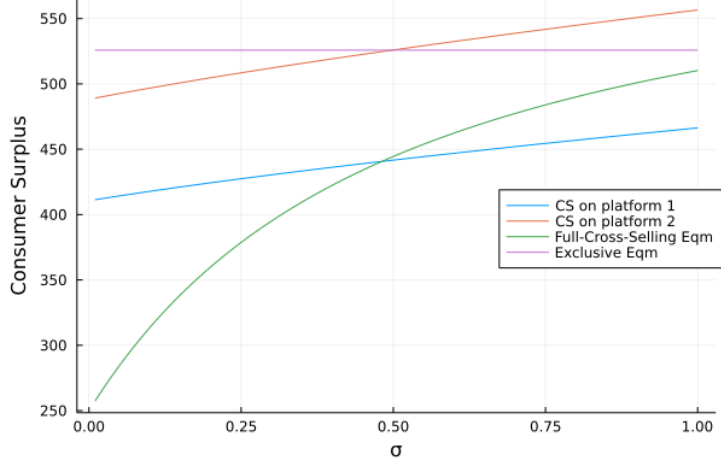


Figure 8: Consumer Surplus Change with Commission

To make this clearer, I draw a 3D graph of the difference in consumer surplus between the two equilibriums (exclusive minus full-cross-selling). The parameters of interest are σ and the console fee, since they are what distinguish the two equilibriums. The difference in consumer surplus is larger with a smaller σ and a larger console fee.

This result may seem counterintuitive, because economists normally expect an exclusive market to have higher prices and thus lower consumer surplus. I discuss in general when this happens in Section 7. A brief intuitive explanation for this finding is collusion. In an exclusive market, the platforms do not interact with each other until the price competition, while in the full-cross-selling market they bargain over the commission and carry it into the price competition. The commission gives them an incentive and at the same time a tool to collude, so that together they drive up the market price and make more profit.

Equilibrium profits also share similar trends across the multihoming and singlehoming cases. Firm 1 benefits from a large σ , while firm 2 prefers a large commission. The key difference is that the exclusive equilibrium now has the lowest profit of all. At the same time, all equilibrium profits decrease with substitutability because of the more intense competition in the market.

5 Stage 2: Nash Bargaining

There is no need for bargaining in an exclusive equilibrium, since the outcome does not depend on σ . In a full-cross-selling equilibrium, both platforms agree on $\sigma = 0$ to maximize the equilibrium profit. This is an interesting contracting mode that differs from both the “outright sale” of Hagiu and Lee (2011), in which the platform obtains both control over pricing and all the sales revenue, and their “affiliation,” in which the content provider sets the price and keeps

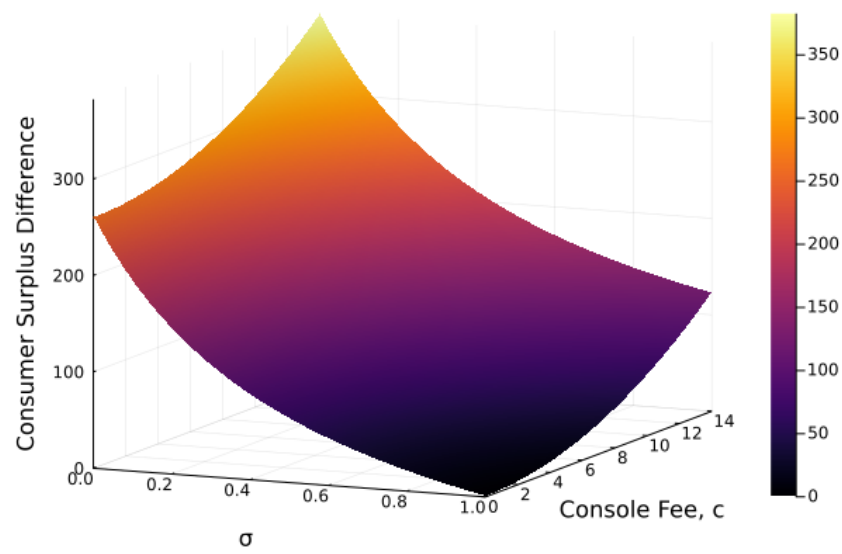


Figure 9: Consumer Surplus Difference

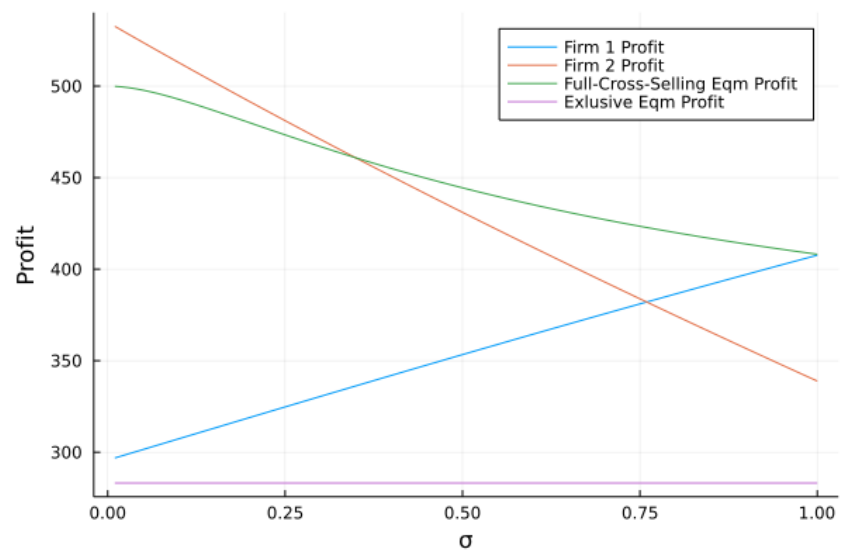


Figure 10: Profit change with commission

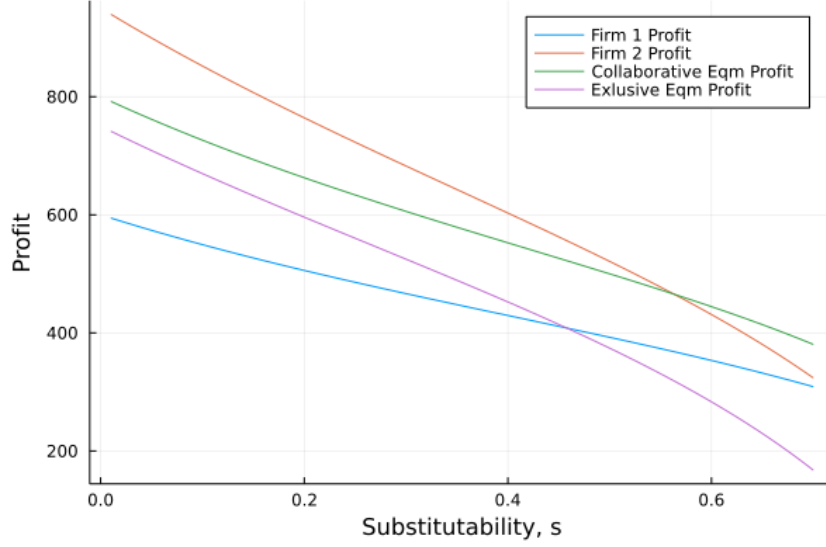


Figure 11: Profit change with substitutability

the sales revenue. Here the content provider, in this case the other platform, sets the price for the content, while the selling platform receives all the sales revenue.

The firms have an incentive to bargain only when exactly one firm wishes to sell its superstar product on the other platform. Disagreement leads to no cross-selling, so the disagreement payoff of both firms is the exclusive equilibrium profit. Following Nash (1953), I define the Nash product $B(\sigma)$ as:

$$B(\sigma) = (\Pi_1(\sigma) - \Pi_{NC})(\Pi_2(\sigma) - \Pi_{NC}) \quad (13)$$

where $\Pi_1(\sigma)$ and $\Pi_2(\sigma)$ are the equilibrium profits of platforms 1 and 2, respectively.

In the semi-cross-selling case, $B(\sigma)$ is strictly concave in $\sigma \in (0, 1)$, so we can always find a $\sigma^S(s)$ for the singlehoming case and a $\sigma^M(s, c)$ for the multihoming case that maximizes the Nash product. Figures 12 and 13 illustrate the negotiated σ with $a = 40$. In the singlehoming case, $\sigma^S(s)$ is decreasing in substitutability and approaches 0. For the multihoming case, Figure 13 draws $\sigma^M(s, c)$ in 3D. As shown in the figure, $\sigma^M(s, c)$ is decreasing in both substitutability and the console fee.

6 Stage 1: Strategy Selection

With the results of stage 2 in hand, I now move back to stage 1 and determine the outcome of the strategy selection game. I set $a = 40$ in the following numerical examples.

Table 1 shows the payoffs of the first-stage game. At stage 1, each firm has two strategies, sell exclusively or cross-sell. Payoffs are determined by Bertrand competition at the negotiated

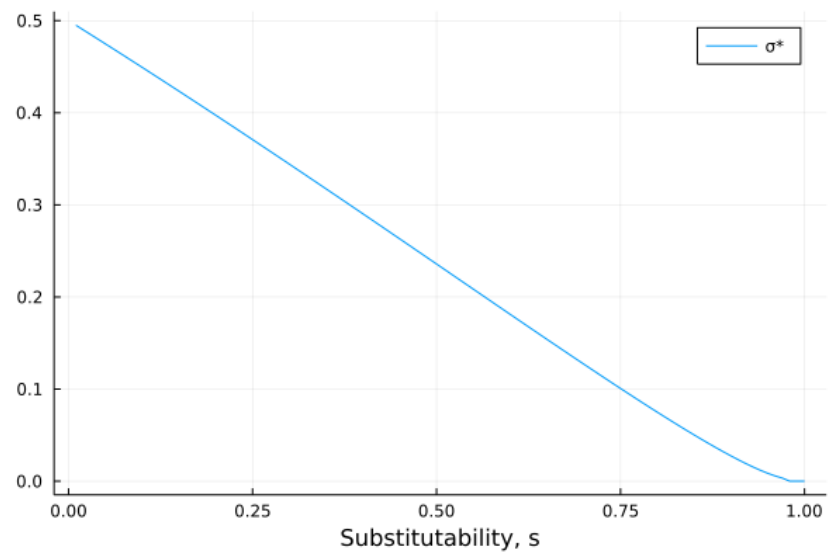


Figure 12: Negotiated σ under singlehoming

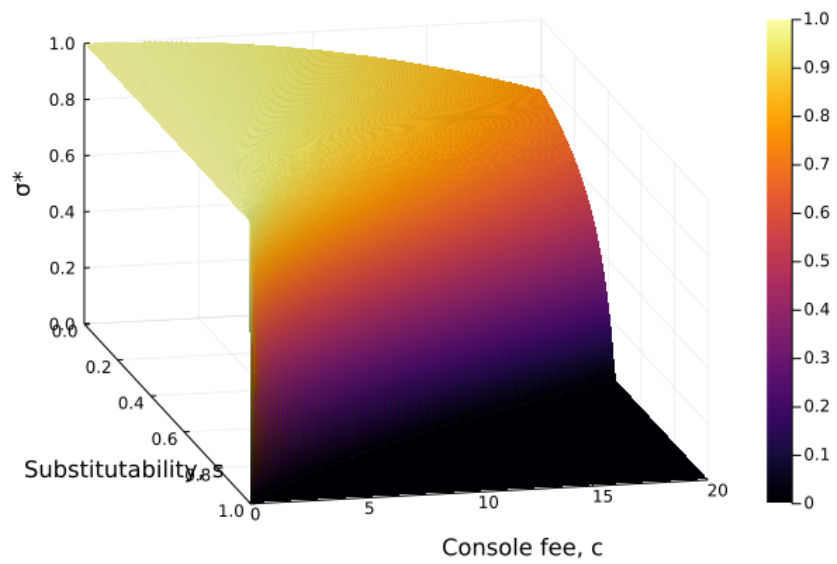


Figure 13: Negotiated σ under multihoming

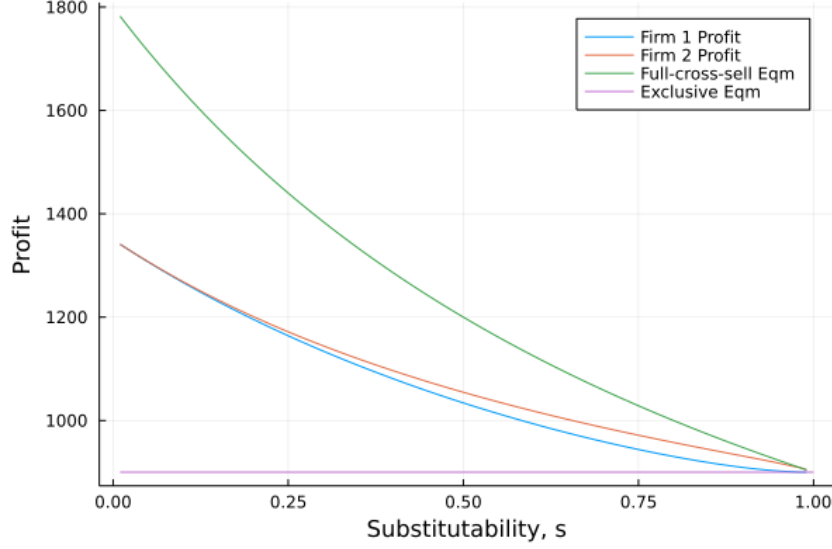


Figure 14: Equilibrium profits under singlehoming

commission rate. Π^E is the exclusive equilibrium profit, Π^C is the full-cross-selling equilibrium profit, and Π^1 and Π^2 are the equilibrium profits of firm 1 and firm 2, respectively, in a semi-cross-selling equilibrium.

	Exclusive	Cross Sell
Exclusive	(Π^E, Π^E)	(Π^2, Π^1)
Cross Sell	(Π^1, Π^2)	(Π^C, Π^C)

Table 1: Payoffs at Stage 1

6.1 Singlehoming Case

Figure 14 illustrates how the equilibrium profits change with substitutability. In the singlehoming case, the equilibrium profit of firm 1 decreases with substitutability when it is the only firm cross-selling or when both firms cross-sell, while the exclusive equilibrium profit does not change with substitutability. The differences among the equilibrium profits shrink as substitutability increases, suggesting that when the products are more alike, the platforms earn less in equilibrium because of stronger market competition.

As Figure 14 shows, for any substitutability $\Pi^E < \Pi^1 < \Pi^2 < \Pi^C$, so both firms adopt the cross-selling strategy.

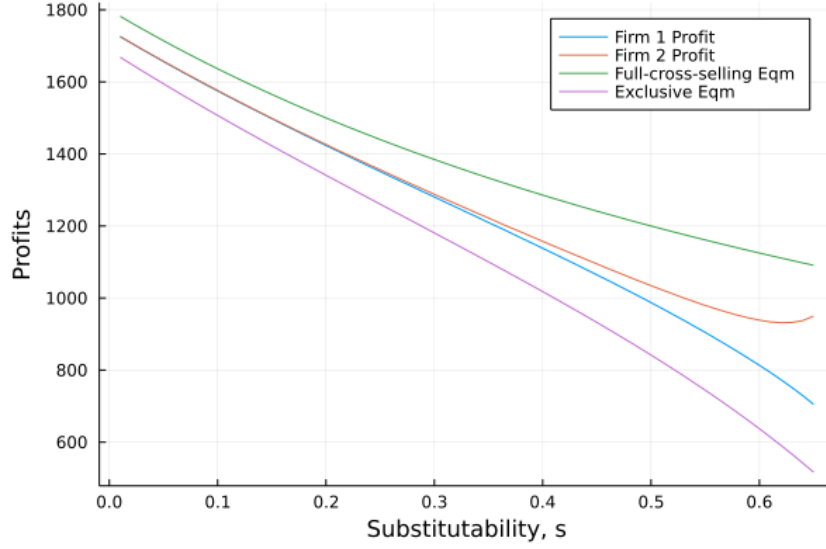


Figure 15: Equilibrium profits under multihoming

6.2 Multihoming Case

The following figure gives a snapshot of how the equilibrium profits of the two firms change with substitutability at a fixed console fee ($c = 15$). As in the singlehoming case, all equilibrium profits decrease with s . The difference is that the exclusive equilibrium profit now also decreases with s because of the decrease in σ^* .

I also draw 3D figures of firm 1's and firm 2's profits in a semi-cross-selling equilibrium. Overall, the two firms' equilibrium profits behave similarly in that both decrease with substitutability. As shown in the figures, firm 1's profit decreases with both substitutability and the console fee, while firm 2's profit increases with the console fee. Firm 2's profit has a non-monotonic relationship with substitutability, as the equilibrium profit shows a slight upward trend when s is very close to its upper limit. I set profits to zero when the combination of s and c leads to negative demand, which explains the cliffs in the profit figures.

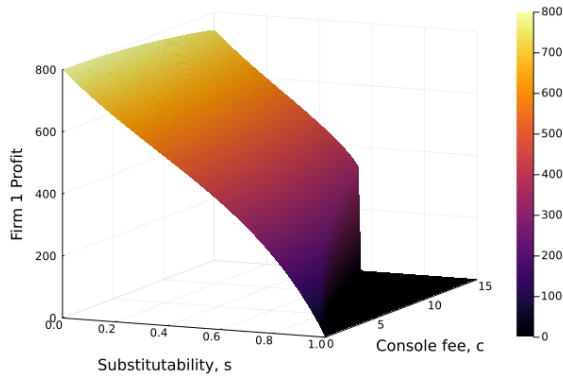


Figure 16: Firm 1 Equilibrium Profit

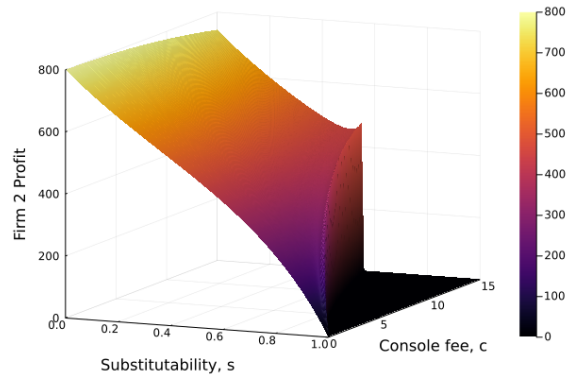


Figure 17: Firm 2 Equilibrium Profit

To better compare the equilibrium profits, I draw a heat map of the difference between the full-cross-selling equilibrium profit and the exclusive equilibrium profit as a function of substitutability and the console fee. As shown in Figure 18, the difference is always positive and increases with both substitutability and the console fee.

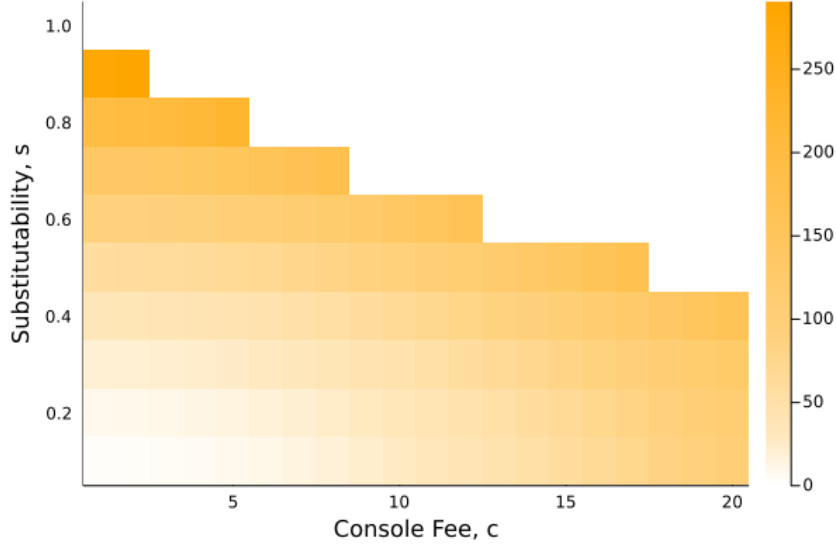


Figure 18: Difference between full-cross-selling and exclusive equilibrium profits

7 General Form Analysis

In this section, I take a closer look at why consumer surplus is lower in the full-cross-selling equilibrium, using a general demand system. In the game, firms maximize profit, so the equilibrium price is always higher than the price that maximizes consumer surplus, and a higher equilibrium price leads to lower consumer surplus and vice versa. Given this, I simplify the problem to examining how the equilibrium prices change with σ and c .

I generalize the model as follows. Let P^X and P^Y denote the prices of products X and Y, respectively. I assume that consumers on both platforms have the same preferences, and their demands for products X and Y are $x(P^X, P^Y)$ and $y(P^X, P^Y)$.

First, I show sufficient conditions for a unique symmetric equilibrium.

Assumption 1. *The demands for the products are symmetric, i.e.,*

$$x(P^X, P^Y) = y(P^Y, P^X) \quad (14)$$

Assumption 2. *(i) The demand for a product is decreasing in its own price and increasing in the rival's price, i.e.,*

$$\frac{\partial x}{\partial P^X} < 0, \quad \frac{\partial x}{\partial P^Y} > 0 \quad (15)$$

(ii) The demand is concave in its own price and in the rival's price, i.e.,

$$\frac{\partial^2 x}{\partial P^X{}^2}, \frac{\partial^2 x}{\partial P^Y{}^2} \leq 0 \quad (16)$$

(iii) The cross partial derivative is non-negative, i.e.,

$$\frac{\partial^2 x}{\partial P^X \partial P^Y}, \frac{\partial^2 y}{\partial P^X \partial P^Y} \geq 0 \quad (17)$$

The profits of both platforms in a full-cross-selling equilibrium can be written as:

$$\begin{aligned} \Pi_1 &= (1 + \sigma) P^X x(P^X, P^Y) + (1 - \sigma) P^Y y(P^X, P^Y) \\ \Pi_2 &= (1 - \sigma) P^X x(P^X, P^Y) + (1 + \sigma) P^Y y(P^X, P^Y) \end{aligned}$$

The first-order conditions are:

$$\frac{\partial \Pi_1}{\partial P^X} = (1 + \sigma) \left(P^X \frac{\partial x}{\partial P^X}(P^X, P^Y) + x(P^X, P^Y) \right) + (1 - \sigma) P^Y \frac{\partial y}{\partial P^X}(P^X, P^Y)$$

$$\frac{\partial \Pi_2}{\partial P^Y} = (1 + \sigma) \left(P^Y \frac{\partial y}{\partial P^Y}(P^X, P^Y) + y(P^X, P^Y) \right) + (1 - \sigma) P^X \frac{\partial x}{\partial P^Y}(P^X, P^Y)$$

Each firm maximizes its profit by choosing its own price, and this process implicitly defines its best response:

$$\begin{aligned} G(P^X, P^Y) &= \frac{\partial \Pi_1}{\partial P^X} = 0 \\ H(P^X, P^Y) &= \frac{\partial \Pi_2}{\partial P^Y} = 0 \end{aligned}$$

Assumption 3. (i) The demand for a product reacts more strongly to its own price than to the rival's price at equilibrium, i.e.,

$$\left| \frac{\partial x}{\partial P^X} \right| \geq \left| \frac{\partial x}{\partial P^Y} \right|, \quad \forall (P^X, P^Y) \text{ s.t. } G(P^X, P^Y) = 0 \quad (18)$$

(ii) The reaction of a product's demand to its own price is stronger than the average reaction of the two products' demands to their rivals' prices at equilibrium, i.e.,

$$\left| \frac{\partial x}{\partial P^X} \right| \geq \frac{1}{2} \left| \left(\frac{\partial x}{\partial P^Y} + \frac{\partial y}{\partial P^X} \right) \right|, \quad \forall (P^X, P^Y) \text{ s.t. } G(P^X, P^Y) = 0 \quad (19)$$

(iii) The marginal effect of a product's own price on its demand reacts more strongly to its own price than to the rival's price, i.e.,

$$\left| \frac{\partial^2 x}{\partial P^X{}^2} \right| \geq \left| \frac{\partial^2 x}{\partial P^X \partial P^Y} \right|, \quad \forall (P^X, P^Y) \text{ s.t. } G(P^X, P^Y) = 0 \quad (20)$$

Proposition 1. *Given Assumptions 1–3, there exists a unique symmetric equilibrium with price P^* .*

Proof. By the implicit function theorem, firm 1's best-response function $P^X = BR_1(P^Y)$ is implicitly defined by $G(P^X, P^Y) = 0$. Its slope is:

$$\frac{\partial P^Y}{\partial P^X} = -\frac{\partial G}{\partial P^X} / \frac{\partial G}{\partial P^Y}$$

where

$$\frac{\partial G}{\partial P^X} = 2(1 + \sigma) \frac{\partial x}{\partial P^X} + (1 + \sigma) P^X \frac{\partial^2 x}{\partial P^{X2}} + (1 - \sigma) P^Y \frac{\partial^2 y}{\partial P^{X2}} < 0$$

$$\frac{\partial G}{\partial P^Y} = (1 + \sigma) \frac{\partial x}{\partial P^Y} + (1 - \sigma) \frac{\partial y}{\partial P^X} + (1 + \sigma) P^X \frac{\partial^2 x}{\partial P^X \partial P^Y} + (1 - \sigma) P^Y \frac{\partial^2 y}{\partial P^Y \partial P^X} > 0$$

Therefore, if at some point (P_1^X, P_1^Y)

$$\frac{\partial x}{\partial P^Y}(P_1^X, P_1^Y) \geq \frac{\partial y}{\partial P^X}(P_1^X, P_1^Y)$$

then by Assumption 3(i),

$$2(1 + \sigma) \left| \frac{\partial x}{\partial P^X} \right| \geq 2(1 + \sigma) \left| \frac{\partial x}{\partial P^Y} \right| > (1 + \sigma) \frac{\partial x}{\partial P^Y} + (1 - \sigma) \frac{\partial y}{\partial P^X}$$

Otherwise, we have

$$\frac{\partial x}{\partial P^Y}(P_1^X, P_1^Y) < \frac{\partial y}{\partial P^X}(P_1^X, P_1^Y)$$

then by Assumption 3(ii),

$$\begin{aligned} 2(1 + \sigma) \left| \frac{\partial x}{\partial P^X} \right| &> 2 \left| \frac{\partial x}{\partial P^X} \right| \\ &> \frac{\partial x}{\partial P^Y} + \frac{\partial y}{\partial P^X} > (1 + \sigma) \frac{\partial x}{\partial P^Y} + (1 - \sigma) \frac{\partial y}{\partial P^X} \end{aligned}$$

Furthermore, by Assumption 3(iii), we have

$$-\frac{\partial G}{\partial P^X} \geq \frac{\partial G}{\partial P^Y} \Rightarrow \frac{\partial P^Y}{\partial P^X} \geq 1$$

In the same manner, firm 2's best-response $P^Y = BR_2(P^X)$ is implicitly defined by $H(P^X, P^Y) = 0$, and we have

$$\frac{\partial P^Y}{\partial P^X} = -\frac{\partial H}{\partial P^X} / \frac{\partial H}{\partial P^Y} < 1$$

Therefore, the two best-response functions can cross at most once, so there is at most one equilibrium.

When $P^Y \rightarrow 0$, the first-order condition of firm 1 becomes

$$(1 + \sigma) \left(P^X \frac{\partial x}{\partial P^X} (P^X, 0) + x(P^X, 0) \right) = 0$$

Since $x(P^X, 0) > 0$ and $P^X \frac{\partial x}{\partial P^X} (P^X, 0) < 0$, we can find a positive P^X that satisfies the first-order condition. Therefore, the best-response function of firm 1 must cross the line $P^Y = P^X$. By the symmetry of Assumption 1, for any given σ the two best-response curves cross at $P^Y = P^X = P^*$ for some P^* .

□

Second, I show what conditions are required for the full-cross-selling equilibrium price to decrease with σ .

Let $f(P^X, P^Y) = P^X x(P^X, P^Y)$ denote a firm's revenue from its own product.

Assumption 4. *At equilibrium, $\frac{\partial f}{\partial P^X}(P^*, P^*)$ is decreasing in the equilibrium price, while $\frac{\partial f}{\partial P^Y}(P^*, P^*)$ is increasing in the equilibrium price.*

This assumption is an extension of Assumption 2, as it further constrains the relationships between the first and second derivatives of the demand function. In terms of economic intuition, it guarantees that as the equilibrium price increases, the marginal cost to a firm of raising its own price increases, and the marginal benefit from a rise in its rival's price also increases.

Proposition 2. *Under Assumptions 1–4, the equilibrium price P^* decreases with σ .*

Proof. Given

$$\frac{\partial x}{\partial P^Y} > 0$$

and

$$(1 + \sigma) \left(P^* \frac{\partial x}{\partial P^X} (P^*, P^*) + x(P^*, P^*) \right) + (1 - \sigma) P^* \frac{\partial x}{\partial P^Y} (P^*, P^*) = 0$$

we must have

$$P^* \frac{\partial x}{\partial P^Y} (P^*, P^*) > 0$$

and

$$P^* \frac{\partial x}{\partial P^X} (P^*, P^*) + x(P^*, P^*) < 0$$

We can solve for σ :

$$\sigma = \frac{P^* \frac{\partial x}{\partial P^Y} (P^*, P^*) + (P^* \frac{\partial x}{\partial P^X} (P^*, P^*) + x(P^*, P^*))}{P^* \frac{\partial x}{\partial P^Y} (P^*, P^*) - (P^* \frac{\partial x}{\partial P^X} (P^*, P^*) + x(P^*, P^*))}$$

Since

$$\begin{aligned} P^* \frac{\partial x}{\partial P^Y} (P^*, P^*) &= \frac{\partial f}{\partial P^Y} (P^X = P^*, P^Y = P^*) \\ P^* \frac{\partial x}{\partial P^X} (P^*, P^*) + x(P^*, P^*) &= \frac{\partial f}{\partial P^X} (P^X = P^*, P^Y = P^*) \end{aligned}$$

the facts that $\frac{\partial f}{\partial P^Y}(P, P)$ is increasing in P and $\frac{\partial f}{\partial P^X}(P, P)$ is decreasing in P guarantee that σ is decreasing in P^* , and vice versa.

Since the firms prefer a higher equilibrium price for more profit, this result suggests that they have an incentive to negotiate and set $\sigma = 0$.

□

Third, I compare the exclusive equilibrium price under multihoming with the full-cross-selling equilibrium price.

The profits of both platforms in an exclusive equilibrium under multihoming can be written as:

$$\begin{aligned}\Pi_1 &= P^X (x_1 + x_2) + cx_2 \\ \Pi_2 &= P^Y (y_1 + y_2) + cy_1\end{aligned}$$

where x_1 and x_2 are the demands for X on the two platforms, and y_1 and y_2 are the demands for Y on the two platforms.

Since the console fee is the only difference on the demand side between the two platforms, we have

$$\begin{aligned}x_1(P^X, P^Y) &= x(P^X, P^Y) \\ x_2(P^X, P^Y) &= x(P^X + c, P^Y)\end{aligned}$$

where x is the general demand function used above.

Proposition 3. *Under Assumptions 1–4, the exclusive equilibrium price is always lower than the full-cross-selling equilibrium price.*

Proof. When $\sigma = 1$, in the full-cross-selling equilibrium we have

$$2 \left(P^* \frac{\partial x}{\partial P^X}(P^*, P^*) + x(P^*, P^*) \right) = 0$$

Under Assumptions 1–4, there exists an exclusive equilibrium with price P^E , which satisfies the first-order condition:

$$x(P^E, P^E) + x(P^E + c, P^E) + P^E \frac{\partial x}{\partial P^X}(P^E, P^E) + (P^E + c) \frac{\partial x}{\partial P^X}(P^E + c, P^E) = 0$$

By Assumption 2, with non-negative c ,

$$x(P^E + c, P^E) \leq x(P^E, P^E), \quad P^E \frac{\partial x}{\partial P^X}(P^E + c, P^E) \leq P^E \frac{\partial x}{\partial P^X}(P^E, P^E)$$

Therefore, if $P^E = P^*$, the left-hand side of the exclusive equilibrium first-order condition would be negative.

By Assumption 4, $x(P, P) + P \frac{\partial x}{\partial P^X}(P, P)$ decreases in P , and thus $P^E < P^*$.

□

8 Conclusion

Vertical integration and exclusivity are both commonly observed in two-sided markets. This paper studies the rationale for platforms' exclusivity strategies and their competitive effects when the platforms are vertically integrated with content providers. I find that the platforms indeed have an incentive to give up exclusivity and supply their rival in a full-cross-selling equilibrium. Compared with the semi-cross-selling and exclusive equilibriums, both firms earn more profit by cross-selling. This result holds whether consumers are singlehoming or multihoming.

It is also interesting that the full-cross-selling equilibrium profit increases with the commission, so both platforms wish to set the commission at 100% of sales revenue. This leads to a new contracting mode in which the content provider has full control over content pricing while the selling platform keeps all the sales revenue. In other words, the seller lets its rival "take the wheel" on the pricing of part of the goods it sells in order to maximize profit.

As for the competitive effect, the full-cross-selling equilibrium generates a consumer surplus higher than that of the singlehoming exclusive equilibrium but lower than that of the multihoming exclusive equilibrium. One potential explanation is that the exclusive equilibrium under singlehoming works as a monopoly market, while under multihoming it works as a competitive market. Bargaining over the commission gives the two platforms a tool to collude, so that they charge a higher price across the whole market and drive consumer surplus down.

Though this paper focuses primarily on the video game industry, it also provides testable implications for other sectors and industries. In the online video industry, many platforms have invested in first-party titles and vertical mergers, and they can choose between streaming exclusively and selling to their rivals under a per-transaction commission. The analysis suggests that exclusivity would lower the market price, especially in a fiercely competitive market. Similarly, the results provide insights into online shopping platforms. This market features both vertical integration and numerical sale. The model predicts that platforms can earn more profit by giving up the exclusivity of their first-party products.

From a policy perspective, the results provide a new lens for understanding the competitive effect of exclusivity. When platforms are vertically integrated with content providers, their dual roles complicate their behavior. The results suggest that exclusivity does not necessarily favor the firms. Rather, under some conditions, firms prefer to give up exclusivity and share the market under a negotiated commission.

This paper also brings a new understanding of two-sided platforms, vertical integration, and collusion. While a rich literature has studied how vertical integration can give upstream or downstream firms the ability to collude, few papers focus on collusion in two-sided markets.

The results show that vertical integration indeed generates incentives for platforms to collude, using the commission as a medium, and can have an anti-competitive effect.

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Appendix A Solution of Semi-cross-selling Equilibrium

Singlehoming Case

Without loss of generality, call the firm that cross-sells firm 1 and the other firm 2. Since consumers are singlehoming, they have access only to product X if they are on platform 1, while they have access to both products if they are on platform 2. The optimization problems of the two groups of consumers are:

$$\begin{aligned} \max_{x_1} U_1(x_1) - P^X x_1 \\ \max_{x_2, y_2} U_2(x_2, y_2) - P^X x_2 - P^Y y_2 \end{aligned}$$

Solving the first-order conditions gives the demands:

$$\begin{aligned} x_1 &= a - P^X \\ x_2 &= \frac{1}{1-s^2} [a(1-s) - P^X + sP^Y] \\ y_2 &= \frac{1}{1-s^2} [a(1-s) - P^Y + sP^X] \end{aligned}$$

Substituting the demands into the firms' profit functions gives

$$\begin{aligned} \Pi_1 &= P^X x_1 + \sigma P^X x_2 \\ &= P^X (a - P^X) + \sigma P^X \frac{1}{1-s^2} [a(1-s) - P^X + sP^Y] \end{aligned}$$

and

$$\begin{aligned} \Pi_2 &= P^Y y_2 + (1-\sigma)P^X x_2 \\ &= P^Y \frac{1}{1-s^2} [a(1-s) - P^Y + sP^X] + (1-\sigma)P^X \frac{1}{1-s^2} [a(1-s) - P^X + sP^Y] \end{aligned}$$

Firm 1 controls only P^X and firm 2 controls only P^Y , so their first-order conditions are:

$$\begin{aligned} \frac{\partial \Pi_1}{\partial P^X} = 0 &\Rightarrow a(1-s)(1+s+\sigma) + \sigma P^Y = 2(1-s^2+\sigma)P^X \\ \frac{\partial \Pi_2}{\partial P^Y} = 0 &\Rightarrow P^Y = \frac{1}{2} [a(1-s) + (2-\sigma)sP^X] \end{aligned}$$

Solving the two conditions simultaneously gives:

$$P^X = \frac{a(1-s)(2+2s+2\sigma+s\sigma)}{[4(1-s^2+\sigma)-s^2\sigma(2-\sigma)]}$$

$$\begin{aligned} P^Y &= \frac{1}{2}a(1-s) + \frac{(2-\sigma)s}{2}P^X \\ &= \frac{1}{2}a(1-s) + \frac{(2-\sigma)s}{2} \frac{a(1-s)(2+2s+2\sigma+s\sigma)}{[4(1-s^2+\sigma)-s^2\sigma(2-\sigma)]} \end{aligned}$$

The numerator of P^X is obviously positive. Its denominator is increasing in σ and decreasing in s , and it is always positive except in the extreme case $\sigma = 0$, $s = 1$, where it goes to 0. P^Y is therefore also positive.

Moreover, it can be shown that

$$P^Y - P^X = -\frac{s\sigma(s+\sigma)a(1-s)}{4(1-s^2+\sigma)-s^2\sigma(2-\sigma)} \leq 0$$

Multihoming Case

As in the previous case, we can use consumers' optimization problems to derive the demand functions.

$$\begin{aligned} x_1 &= \frac{1}{1-s^2} [a(1-s) + sP^Y + sc - P^X] \\ x_2 &= \frac{1}{1-s^2} [a(1-s) + sP^Y - P^X] \\ y_1 &= \frac{1}{1-s^2} [a(1-s) + sP^X - c - P^Y] \\ y_2 &= \frac{1}{1-s^2} [a(1-s) + sP^X - P^Y] \end{aligned}$$

Substituting the demands into the firms' profit functions, we have:

$$\begin{aligned} \Pi_1 &= P^X x_1 + \sigma P^X x_2 \\ &= P^X \frac{1}{1-s^2} [a(1-s) + sP^Y + sc - P^X] + \sigma P^X \frac{1}{1-s^2} [a(1-s) + sP^Y - P^X] \end{aligned}$$

$$\begin{aligned}
\Pi_2 &= (1 - \sigma)P^X x_2 + P^Y(y_1 + y_2) + cy_1 \\
&= (1 - \sigma)P^X \frac{1}{1 - s^2} [a(1 - s) + sP^Y - P^X] + P^Y \frac{1}{1 - s^2} [2a(1 - s) + 2sP^X - 2P^Y - c] \\
&\quad + c \frac{1}{1 - s^2} [a(1 - s) + sP^X - c - P^Y]
\end{aligned}$$

Solving the first-order conditions gives:

$$\begin{aligned}
P^X &= \frac{1}{2} \left[a(1 - s) + \frac{sc}{1 + \sigma} + sP^Y \right] \\
P^Y &= \frac{1}{8 - s^2(3 - \sigma)} \left\{ (1 - s)[as(3 - \sigma) + 4a] - c(4 - \frac{3 - \sigma}{1 + \sigma}s^2) \right\}
\end{aligned}$$